In the continuations of these Floras we have the same standard of excellence to which preceding volumes have accustomed us, and which we therefore look for in publications coming from Kew. Their issue will be a boon not only to the professed botanical world, but also to all those who are interested in the many plants now known in, and still coming into cultivation from, Africa; and they should give a great stimulus to the further investigation of the vegetation of Africa and to the introduction of interesting and beautiful plants to the horticulture of the world.

No one looking at these volumes can fail to notice that their production at Kew, where a collection of living plants in a garden is associated with one of dried specimens in a herbarium, gives additional value to them. The necessity of the latter as a guide to the accurate determination of the nomenclature in a scientific garden is apparent; the service of the former as an adjunct to the herbarium by affording means for the study of the living plants in cases where the dried specimen can seldom be satisfactory is clearly brought out in the account of the groups of succulent monocotyledons treated of in these Floras. If all descriptive botanists were able, as is possible at Kew, to look at the dry bones of the plants with which they deal with some consideration of the form that clad them when alive, we should be spared much of that prolific synonymy which is the bane of the systematist. It is the possession of the finest collection of living plants along with a like one of dried specimens, through which it can contribute as in these Floras to the advance of our knowledge of the vegetation of the globe, that gives Kew an absolutely unique position as the leading botanical institution of the world, a position it has achieved in little over fifty years through the scientific ability and remarkable administrative capacity of its successive Directors, Sir William Hooker, Sir Joseph Hooker, and Sir William Thiselton-Dyer.

STATISTICAL METHODS APPLIED TO BIOLOGY.

Die Methode der Variationsstatistik. Von Georg Duncker. Pp. 75, with 8 figures in text. (Leipzig: Wilhelm Engelmann.)

THIS pamphlet, a reprint from the *Archiv für Entwickelungsmechanik*, is an attempt to render the formulæ and results of the statistical method somewhat more accessible to German biologists than they are, for example, in Prof. Karl Pearson's original papers. In the first part a complete outline is given of the fitting of frequency curves, normal or skew, to observed statistics, and in the second part a similar outline of the theory of correlation. The whole of this extensive ground is covered, however, in some sixty octavo pages, necessitating a degree of compression too great for satisfactory results. Proofs are necessarily almost wholly omitted, several difficulties likely to occur to beginners are slurred over, and there is more than one absolute blunder.

If $y = \phi(x)$ be any frequency curve, the frequency of deviations lying between $x - \frac{1}{2}c$ and $x + \frac{1}{2}c$ is given by the integral of the frequency-function $\phi(x)$ between those limits. If, and only if, c be very small, we may replace this integral by the product $y.c$ to a close degree of approximation. Hence, in any practical case of recording the distribution of frequency where we have to choose an arbitrary unit of grouping c, this should always be made as small as possible. If this be done, and if the number of observations be large, the observed frequency polygon closely approximates to a continuous curve, and the element of area round any ordinate differs very slightly from $y.c$. Moreover, the process of obtaining the moments of the observed frequency polygon by treating the observed frequencies as isolated loads, then differs very slightly in result from the process of continuous integration by which the moments of the theoretical curve were calculated. But if the element of grouping be not small, the element of area round y may differ very sensibly from $y.c$, and the process of calculating moments by treating group frequencies as isolated loads is not even a rough approximation to continuous integration. Hence Prof. Pearson's original preference of the moments of the trapezia system (*Phil. Trans.* A, 1895, "On Skew Variation," &c.), and Mr. W. F. Sheppard's papers on moment calculation (*Proc. Lond. Math. Soc.*, vol. 29, and *Journal Roy. Statistical Soc.*, vol. 60, 1897). This difficulty, due to the grouping, is entirely passed over by Herr Duncker. A series of observations giving only five base elements c is fitted without remark to a normal curve (Fig. 2). In every case the ordinates of the fitted curve are calculated only for the abscissæ of the observed ordinates, their tops are joined up, and the polygon so obtained called the "theoretical frequency polygon," as in Figs. 1, 2, 3—a procedure of somewhat dubious use in any case, and quite illegitimate where the elements are as large as in Fig. 2. If the author had not missed this fundamental point he would not, perhaps, have been so puzzled by Prof. Pearson's use of first one and then another method of calculating moments. It is a pity that all the arithmetical examples given of fitting frequency curves refer to cases of discontinuous variation, as these are naturally the material least suitable for representation by continuous curves.

There seems a corresponding lack of clearness in some fundamental points of the theory of correlation. The various formulæ for correlation coefficient, regressions, &c., are given, but the author nowhere clearly points out their meanings and limitations in cases of non-normal correlation. It is not noted that for complete independence the condition $r = o$ is, in general, necessary but *not sufficient*. The values of partial correlations are given, but it is not noted that only in normal correlation (so far as we know) is the partial correlation the same for every array. The correction given on p. 48 for reducing the product sum about an arbitrary pair of axes to the product sum about the mean is surely absolutely wrong; the different sums given are all of different dimensions. If S_0 be the value of the sum for axes through the mean, S_1 its value for the arbitrary axes

$$S_0 = S_1 - N.\,\bar{x}\,\bar{y},$$

where N is the number of observations and $\bar{x}\,\bar{y}$ are the coordinates of the mean with reference to the arbitrary axes. Of course this expression, and method of getting S_0, is quite well known, not novel as Herr Duncker seems to think.

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Again, complete correlation does not subsist only when "every single deviation of the one characteristic corresponds to a precisely equal deviation of the other" (p. 43), but whenever the deviations are in any constant proportion. This definition, moreover, ought only to be held to apply to the case where the means of arrays lie on straight lines.

We do not like the definition of correlation as a "Beziehung . . . welche bewirkt. . . ." This is not a statistical definition, and confusion arises if the word be used carelessly, sometimes in one sense and sometimes in another. The definition of the correlation coefficient on p. 54 is much better; but why does the author call it a "morphological definition"? It is purely statistical.

The statement that it is mainly correlation which maintains the type ("die Korrelation ist es hauptsächlich, welche den Typus einer Formengemeinschaft aufrecht erhält") is a very pretty error; possibly due to the fact that biologists use correlation in a more "intensive" sense than statisticians do. Statistical correlation has absolutely nothing whatever to do with the maintenance of type. The type is described by the coordinates of the mode. If X_0 be the modal size of any organ in a parent, Y_0 the modal size of the same organ in the offspring, the "type is maintained," or constant, whenever

$$X_0 = Y_0,$$

and this relation is quite independent of the correlation. The correlation between parent and offspring might be absolutely zero, *i.e.* every single parent's offspring might be a fair sample of the whole population, and yet the type might remain absolutely fixed; or, on the other hand, parent and offspring might be perfectly correlated, and yet the type change entirely. This is at least *formally* possible. Thus, in the extreme case of alternating generations A B A B . . ., there might in the statistician's sense be perfect correlation—perfect inheritance—between A and B, although A and B differ absolutely.

Of course in a work of the present kind, written chiefly for drawing attention to the work of others, one does not look for much that is original. There is a curious approximate relation given by the author between geometric mean, arithmetic mean, and standard deviation (p. 38), a relation discovered empirically and given without proof. If

G = geometric mean,
M = arithmetic mean,
σ = standard deviation,

then *approximately* in many cases

$$\sigma^2 = M^2 - G^2.$$

The relation depends solely on all deviations being small compared with the mean, and admits of a simple algebraical proof.

Amongst small points we have marked, we would like the term "individual" variation suppressed, as it is frequently misleading, and surely not equivalent to "spontaneous"; the word "variant" seems to us unnecessary and misleading in the case of continuous variation where arbitrary groupings are used; the distinction between "Rasse" and "Formeneinheit" (p. 17) (". . . erstere nothwendig in mehreren Merkmalen, letztere in einem einzigen differiren") is surely inadequate; a preliminary calculation of the mean before calculating the moments of a frequency distribution (p. 18) is quite unnecessary, as the mean is given by the first moment; and the author is unfortunately in error in ascribing to the present writer (p. 52) the extension of the formulæ of correlation to several variables.

We regret that this notice has had to be for the most part fault-finding, as the author has undertaken a useful and somewhat thankless task, and we believe that, notwithstanding our criticisms, the pamphlet will be useful in extending a knowledge of the statistical method in Germany. There is a bibliography of 111 items at the end of the pamphlet, a feature which will render it useful to English readers as well as German. We are, of course, in sympathy with the author's aim, and hope he may have the opportunity of revising some of the points we have noted in a second issue. G. U. Y.

TEXT-BOOKS OF PHYSICS.

Physics, Experimental and Theoretical. By R. H. Jude and H. Gossin. Pp. xiii + 926. (London: Chapman and Hall, Ltd., 1899.)

THE increasing study of science in schools has been the cause of a considerable crop of text-books of elementary physics, but there is still the want of a more advanced book on the subject. This want Mr. R. H. Jude has endeavoured to supply, and as far as can be judged by a glance through his book, supplemented by a more careful examination of a few chapters, he has succeeded in giving us what promises to be a very useful work both to teachers and students. Experience only can show whether he has hit on the right standard of difficulty, and whether the learner will find the explanations sufficiently clear and complete; but there seems no reason to doubt it, considering that the work is an English adaptation of a book by Prof. Gossin, which is apparently much used in France.

Originally intended to be a translation, the volume before us contains many new articles and chapters, and the translated portions have been amplified. The first volume treats of mechanics, heat and sound. The following remarks are not intended to be special criticisms of this particular book, but rather are suggested by it and put down as matters for consideration, being of general interest to teachers of science.

It seems a little doubtful to me how far a book which contains a somewhat advanced treatment of experimental physics should enter into questions of elementary mechanics. It is impossible to believe that a student who can follow the method of treatment given in the chapters on heat in this volume should not be familiar with the parallelogram of forces, and the construction of the common pump. Some portions of dynamics, such as moments of inertia, must, of course, be included, and it may be argued that it is better to present a complete than a partial statement of mechanical principles. This is true, and of course a good deal might be said about the parallelogram of forces and velocity that is worth reading at any stage in a student's career, but what strikes me in this volume is that the standard of treatment does not quite correspond,

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